Solutions to JEE Advanced Revision Test - 2 | Paper-1 | JEE 2024

PHYSICS

SECTION-1

1.(AD) Area of the loop, $A = \pi a^2$

Resistance of the loop, $R = \dfrac{\rho(2\pi a)}{\left(\dfrac{\pi d^2}{4}\right)} = \dfrac{8\rho a}{d^2}$

Instantaneous induced current, $I = \dfrac{A}{R}\dfrac{dB}{dt} = \dfrac{\pi B_0 \omega a d^2}{8\rho}\cos(\omega t)$

Therefore, maximum current, $I_M = \dfrac{\pi B_0 \omega a d^2}{8\rho}$

Instantaneous rate of heat dissipation,

$$H = I^2 R = \left(\frac{\pi B_0 \omega a d^2}{8\rho}\cos(\omega t)\right)^2 \left(\frac{8\rho a}{d^2}\right) = \frac{\pi^2 B_0^2 \omega^2 a^3 d^2}{8\rho}\cos^2(\omega t)$$

Since the average value of $\cos^2(\omega t)$ over many cycles is 1/2, $H_{AV} = \dfrac{\pi^2 B_0^2 \omega^2 a^3 d^2}{16\rho}$

2.(ACD) Resistance of the portions PS and SQ is

$R_{PS} = \left(\dfrac{PS}{PQ}\right)(40) = 30\ \Omega$ and $R_{SQ} = \left(\dfrac{SQ}{PQ}\right)(40) = 10\ \Omega$

Potential difference between P and S is $V_P - V_S = (i_1 - i_2)R_{PS} = (0.8)\left(\dfrac{45}{2}\right) = 24$ V

Since it is ideal, the EMF of the battery E_1 must be equal to V_{PQ}

So, $E_1 = V_{PQ} = V_P - V_Q = (i_1 - i_2)R_{PS} + i_1 R_{SQ} = (0.8)(30) + (1.8)(10) = 42$ V

Starting from P and applying KVL in the small loop, going clockwise:

$-(i_1 - i_2)(30) + E_2 + i_2 r_2 = 0 \Rightarrow -(0.8)(30) + 20 + r_2 = 0 \Rightarrow r_2 = 4\ \Omega$

Since $E_2 < E_1$, current through the branch containing

E_2 can be made zero by changing length PS.

3.(BCD) Field at *S* due to side *PQ* is in the +Z direction.

The perpendicular distance of *S* from *PQ*, $r_{PQ} = \dfrac{a\sqrt{3}}{2}$

So, field due to *PQ*, $B_{PQ} = \dfrac{\mu_0 I}{4\pi r_{PQ}}\left(\cos 30^\circ + \cos 120^\circ\right) = \dfrac{\mu_0 I}{4\pi a}\left(1 - \dfrac{1}{\sqrt{3}}\right)$

Field at *S* due to side *RP* is also in the +Z direction and has the same magnitude due to symmetry.

Now, field at *S* due to side *QR* is in the –Z direction.

The perpendicular distance of *S* from *QR*,

$$r_{QR} = \frac{a\sqrt{3}}{2}$$

So, field due to QR, $B_{QR} = \frac{\mu_0 I}{4\pi r_{QR}}\left(\cos 60^\circ + \cos 60^\circ\right) = \frac{\mu_0 I}{2\sqrt{3}\pi a}$

Net field at S, $\vec{B}_{net} = \vec{B}_{PQ} + \vec{B}_{QR} + \vec{B}_{RP} = \left[\frac{\mu_0 I}{2\pi a}\left(\frac{2}{\sqrt{3}} - 1\right)\right]\left(-\hat{k}\right)$

SECTION-2

4.(B) Resistance of the shell, $r = \int_R^{3R}\left(\frac{\rho}{2\pi x L}\right)dx = \left(\frac{\rho}{2\pi L}\right)\log_e(3)$

So, the current through the shell, $I = \frac{V}{r} = \frac{2\pi L V}{\rho \log_e(3)}$

Electric field at a point at a distance $2R$ from the axis of the shell is

$$E = \rho j = \rho\left(\frac{i}{2\pi(2R)L}\right) = \frac{V}{2R\log_e(3)}$$

5.(A) We know that $I = +\frac{dQ}{dt} \Rightarrow \int_0^Q dQ = \int_0^t \left(I_0 \cos(\omega t)\right) dt \Rightarrow Q(t) = \frac{I_0}{\omega}\sin(\omega t)$

Remember that in a series LCR (or RC) circuit, the charge on the capacitor is always behind the current in the circuit by a phase angle $\frac{\pi}{2}$. This does not depend on the values of R, L or C or the frequency.

Hence the given angular frequency $\omega = \frac{1}{RC}$ is of no consequence to the question asked.

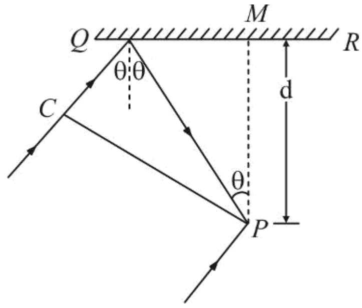
6.(B) Taking a Gaussian surface of radius x (such that $\frac{R}{2} \le x \le R$), and applying Gauss law,

$$E\left(4\pi x^2\right) = \frac{Q_{encl}}{\varepsilon_0}$$

Now, $Q_{encl} = \int_{R/2}^{x} \rho_0\left(\frac{r}{R}\right)\left(4\pi r^2 dr\right) = \frac{\pi\rho_0}{R}\left(x^4 - \frac{R^4}{16}\right)$

Therefore, $E = \frac{\rho_0}{4\varepsilon_0 R}\left(x^2 - \frac{R^4}{16x^2}\right)$

7.(B) In ΔOPM, $\frac{OM}{OP} = \cos\theta \Rightarrow OP = \frac{d}{\cos\theta}$



In ΔCOP, $\cos 2\theta = \frac{OC}{OP} \Rightarrow OC = \frac{d\cos 2\theta}{\cos\theta}$

Path difference between the two rays reaching P is

$$CO + OP + \frac{\lambda}{2} = \frac{d\cos 2\theta}{\cos\theta} + \frac{d}{\cos\theta} + \frac{\lambda}{2}$$

$$= \frac{d}{\cos\theta}\left(\cos 2\theta + 1\right) + \frac{\lambda}{2} 2d\cos\theta + \frac{\lambda}{2}$$

Path difference between the two rays reaching P is $n\lambda$,

$$\therefore \quad 2d\cos\theta + \frac{\lambda}{2} = n\lambda \quad \Rightarrow \quad 2d\cos\theta = \left(n - \frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad 2d\cos\theta = \frac{(2n-1)}{2}\lambda \quad \Rightarrow \quad \cos\theta = \frac{(2n-1)}{2}\frac{\lambda}{2d}$$

For $\quad n = 1, \cos\theta = \frac{\lambda}{4d}$

SECTION-3

8.(B) Let the velocity of the particle when it enters the magnetic field be v

We know that

Change in kinetic energy of the particle = Work done on the particle by the electric field

Therefore, $\quad \frac{1}{2}mv^2 = qEd \quad \Rightarrow \quad v = \sqrt{\frac{2qEd}{m}}$

Now, the radius of the circular path of the particle in the magnetic field,

$$r = \frac{mv}{qB} = \frac{1}{B}\sqrt{\frac{2mEd}{q}}\ ; \text{ So,} \quad y_0 = 2r = \frac{2}{B}\sqrt{\frac{2mEd}{q}} = \left(\frac{8mEd}{qB^2}\right)^{1/2}$$

9.(D) Let the linear momentum of the particle before the split be p_0

Then, radius of the particle in the field if it did not split, $r_0 = \frac{p_0}{qB}$

So, $\quad y_0 = 2r_0 = \frac{2p_0}{qB}$

Now, radii of paths of particles after splitting are:

$$r_1 = \frac{(p_0/4)}{(3q/2)B} = \frac{p_0}{6qB}; \qquad r_2 = \frac{(3p_0/4)}{(q/2)B} = \frac{3p_0}{2qB}$$

Since the particles curve in opposite directions,

$$|y_1 - y_2| = 2(r_1 + r_2) = 2\left(\frac{p_0}{6qB} + \frac{3p_0}{2qB}\right) = \frac{10p_0}{3qB} = \frac{5}{3}y_0$$

10.(B) **11.(D)**

In both questions we need to apply Lenz law. Also, remember that in each case, the magnetic force on the sides of the loop which are perpendicular to the wire gets cancelled mutually, and so has no bearing on the direction of the net magnetic force on the loop.

10. If I is increased, the flux into the plane through the loop increases, so to counter this change, an anti-clockwise current is induced. We can deduce that this results in a magnetic force away from the wire

If I is decreased, the flux into the plane through the loop decreases, so to counter this change, a clockwise current is induced. We can deduce that this results in a magnetic force towards the wire

11. If the loop is moved away, the flux into the plane through the loop decreases, so to counter this change, a clockwise current is induced. We can deduce that this results in a magnetic force towards the wire

If the loop is moved towards the wire, the flux into the plane through the loop increases, so to counter this change, an anti-clockwise current is induced. We can deduce that this results in a magnetic force away from the wire

Hence, in both cases, the magnetic force on the loop is opposite to its velocity

SECTION-4

1.(2) Due to the rotation of the loop, a current is induced in it, and due to this induced current, the magnetic field applies a torque on the loop. We can check easily that this torque will oppose the rotation of the loop

At an instant when the loop has rotated by angle θ from its initial position, the flux through it is

$$\Phi = BL^2 \cos\theta$$

Therefore, at this instant the induced current is $i = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{BL^2}{R}\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{BL^2\omega}{R}\sin\theta$

Here, ω is the instantaneous angular velocity of the loop

So, the instantaneous magnetic moment of the loop is $M = iL^2 = \frac{BL^4\omega}{R}\sin\theta$

And, the instantaneous torque on it is $\tau = MB\sin\theta = \frac{B^2L^4\omega}{R}\sin^2\theta$

Let the moment of inertia of the loop about the Y-axis be I_Y

Then, if the angular velocity of the loop immediately after it is given the angular impulse is ω_0,

$$J = I_Y\omega_0$$

Also, since we know that the magnetic torque on the loop opposes its angular motion,

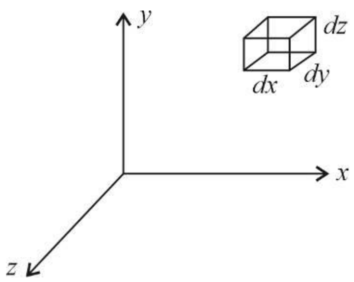
$$\tau = -I_Y\left(\omega\frac{d\omega}{d\theta}\right) \qquad (\left(\omega\frac{d\omega}{d\theta}\right) \text{ is the angular acceleration of the loop})$$

$$\Rightarrow \quad I_Y\left(\omega\frac{d\omega}{d\theta}\right) = -\frac{B^2L^4\omega}{R}\sin^2\theta$$

$$\Rightarrow \quad I_Y\int_{\omega_0}^{0} d\omega = -\frac{B^2L^4}{R}\int_0^{\pi}\sin^2\theta\, d\theta$$

$$\Rightarrow \quad I_Y\,\omega_0 = \frac{B^2L^4}{R}\left(\frac{\pi}{2}\right) \quad \Rightarrow \quad J = \left(\frac{\pi}{2}\right)\frac{B^2L^4}{R}$$

2.(4)



$$\phi = \left(E_{(x+dx)} - E_x\right)dydz + \left(E_{(y+dy)} - E_y\right)dxdz + \left(E_{(z+dz)} - E_z\right)dxdy = \frac{\rho(dxdydz)}{\epsilon_0}$$

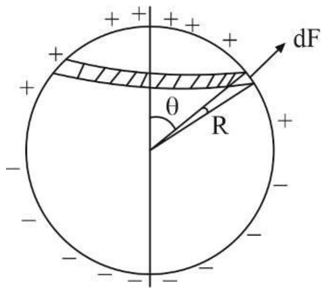
$$\Rightarrow \quad \frac{\rho}{\epsilon_0} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \quad \Rightarrow \quad \rho = \epsilon_0\,(2x - 2y + 2) = 4\epsilon_0$$

3.(135) $Q = (226.005 - 222.000 - 4.00)931\ MeV$; $= (0.005)931\ MeV = 4.655\,MeV$

E_T = Total Energy available $= \dfrac{A}{A-4} \times E_\alpha$; $\dfrac{226}{222} \times 4.44 = 4.52\,MeV$

$E_r = Q - E_T = 4.655 - 4.52\,MeV = 135\,KeV$

4.(4)



$dF = \dfrac{\sigma E dA}{2}$; $E = \dfrac{\sigma}{\epsilon_0}$

$dA = (2\pi R \sin\theta) R d\theta$

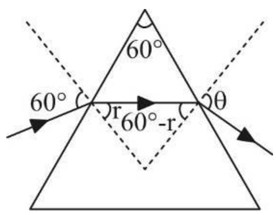
$$F_{net} = \int dF \cos\theta = \int_0^{\pi/2} \frac{\pi\sigma^2 R^2}{\epsilon_0} \sin\theta\cos\theta d\theta = \frac{\pi\sigma_0^2 R^2}{4\epsilon_0}$$

5.(2) For the first capacitor, $\dfrac{1}{C_{eq}} = \dfrac{\left(\frac{d}{3}\right)}{K\varepsilon_0 A} + \dfrac{\left(\frac{2d}{3}\right)}{4K\varepsilon_0 A} = \dfrac{d}{2K\varepsilon_0 A} \Rightarrow C_{eq} = \dfrac{2K\varepsilon_0 A}{d}$

For the second capacitor, $C_{eq} = \dfrac{K_0\varepsilon_0 A}{d} \Rightarrow \dfrac{K_0}{K} = 2$

6.(2) $\sin 60 = n\sin r$ … (i)

$\sin\theta = n\sin(60 - r)$ … (ii)



Differentiating equation (ii).

$$\cos\theta\frac{d\theta}{dt} = -n\cos(60-r)\frac{dr}{dt} + \sin(60-r)$$

Differentiating equation (i).

$n\cos r\dfrac{dr}{dt} + \sin r = 0$; $\cos\theta\dfrac{d\theta}{dt} = -n\cos(60-r)\left(\dfrac{-\tan r}{n}\right) + \sin(60-r)$

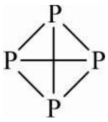
$$\frac{d\theta}{dt} = \frac{1}{\cos\theta}\left[+\cos(60-r)\tan r + \sin(60-r)\right]$$

$$\frac{d\theta}{dt} = \frac{1}{\cos 60}(\cos 30 \times \tan 30 + \sin 30) = 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

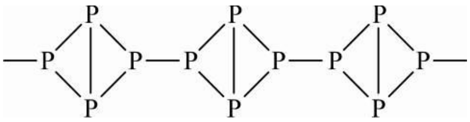
CHEMISTRY

SECTION-1

1.(ABCD) White phosphorous is most reactive form of phosphorous due to strain in rings.



Red phosphorous is linear polymeric form of P_4 units.



Black phosphorous is a network solid hence it is thermodynamically most stable form.

For white phosphorous, $\Delta_f H^\circ$ is zero.

2.(ACD)

(A) $NaNO_2 + NH_4Cl \xrightarrow{\Delta} N_2 + NaCl + H_2O$

(B) Mix. Of HNO_2 and $NaNO_2$ is acidic buffer

(C) $NH_3 + CuO \longrightarrow Cu + H_2O + N_2$

(D) $KMnO_4 + NH_3 \xrightarrow[\text{Medium}]{\text{Neutral}} MnO_2 + N_2 + KOH + H_2O$

3.(ABC)

$$CH_3-\underset{\underset{O}{\|}}{C}-OH \xrightarrow{NaOH} \xrightarrow{\text{electrolysis}} CH_3-CH_3 \xrightarrow[h\nu]{Br_2} CH_3-CH_2-Br$$

$$\downarrow \text{alc.KOH, } \Delta$$

$$\underset{(B)}{CH_3-CH_2-CH_2OH} \xleftarrow{Ni/H_2} \underset{(A)}{CH_3-CH_2-\underset{\underset{O}{\|}}{C}-H} \xleftarrow[Co_2(CO)_8]{CO+H_2} CH_2=CH_2$$

SECTION-2

4.(C)

$$Y_A = \frac{P_A^\circ X_A}{P_A^\circ X_A + P_B^\circ X_B}$$

$$\frac{1}{Y_A} = 1 + \frac{P_B^\circ}{P_A^\circ}\frac{X_B}{X_A}$$

$$\frac{1}{Y_A} = 1 + \frac{P_B^\circ}{P_A^\circ}\frac{(1-X_A)}{X_A} = \left(1 - \frac{P_B^\circ}{P_A^\circ}\right) + \frac{P_B^\circ}{P_A^\circ}\cdot\frac{1}{X_A}$$

$$\left(\frac{1}{Y_A} - \left(1 - \frac{P_B^\circ}{P_A^\circ}\right)\right)\frac{P_A^\circ}{P_B^\circ} = \frac{1}{X_A}$$

$$\frac{1}{X_A} = \frac{P_A^\circ}{P_B^\circ}\cdot\frac{1}{Y_A} + \left(\frac{P_B^\circ - P_A^\circ}{P_B^\circ}\right)$$

5.(D) $\frac{P_A^\circ - P_S}{P_S} = \frac{n}{N}$

$\frac{89.78-89}{89} = \frac{(2/Mo)}{(100/78)} \Rightarrow M_o = 178 g/mol$

%C by mass of C_XH_Y hydrocarbon $= \frac{12x}{178}\times 100 = 94.4$

$x = 14$ Formula of hydrocarbon $= C_{14}H_{10}$

6.(D) $2.5\times 10^{-5} = \frac{1}{100\times 60}\ell n\frac{[SO_2Cl_2]_o}{[SO_2Cl_2]_t}$

$0.15 = \ell n\frac{[SO_2Cl_2]_o}{[SO_2Cl_2]_t} \Rightarrow \frac{[SO_2Cl_2]_o}{[SO_2Cl_2]_t} = 1.16$

% SO_2Cl_2 decomposed $= \frac{[SO_2Cl_2]_o - [SO_2Cl_2]_t}{[SO_2Cl_2]_o}\times 100 = \left(1-\frac{1}{1.16}\right)\times 100 = 13.8\%$

7.(C) $\frac{x}{m} = KP^{\frac{1}{n}}$

$0.2 = K\times(4)^{\frac{1}{n}}$ …(1)

$0.5 = K\times(25)^{\frac{1}{n}}$ …(2)

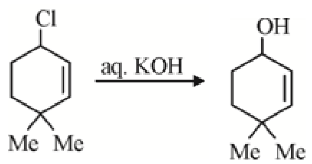
$\frac{2}{5} = \left(\frac{4}{25}\right)^{\frac{1}{n}} \Rightarrow n = 2, K = 0.1$

$\frac{x}{m}$ at 36 bar pressure $= 0.1\times(36)^{\frac{1}{2}} = 0.6$

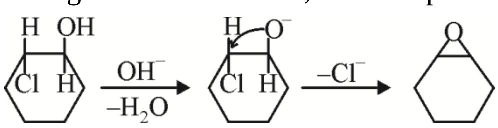
Mass of $N_2(g)$ adsorbed per g of iron = 0.6g

Moles of $N_2(g)$ adsorbed per g of iron $= \frac{0.6}{28} = \frac{3}{140}$

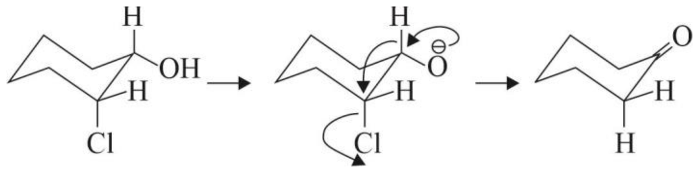
SECTION-3

8.(D) 

9.(D) In the reactant (I), the Cl and OH groups are anti-coplanar. So, the nucleophilic O^- displaces Cl^- through a backside attack, to form epoxide. This is an example of neighbouring group participation.



In reactant (II) both Cl and OH are syn to each other. In this conformation Cl and H (of CHOH group) are diaxial which leads to elimination to give to ketone (cyclohexanone).



10.(D) $NCl_3 + 3H_2O \longrightarrow NH_3 + 3HOCl;\ 2HOCl \longrightarrow \underset{\text{anhydride of HOCl}}{Cl_2O} + H_2O$

11.(C) coloured matter + $SO_2 + H_2O \longrightarrow$ colourless matter + SO_3

colourless matter $\xrightarrow{\text{air}}$ coloured matter

SECTION-4

1.(160) $\Delta T_f = K_f.m$

$$26.84 - 25.64 = 8 \times \frac{(2.4 \times 10^{-3})}{M_o \times (100 \times 10^{-3})} \times 1000$$

$$M_o = 160 g/mol = 0.16 kg/mol$$

2.(90) $H_2 + 2Ag^+ \rightleftharpoons 2Ag + 2H^+$

$$E_{cell} = 0.5 = (0.8 - 0) - \frac{0.06}{2}\log_{10}\frac{1}{1 \times [Ag^+]^2}$$

$\Rightarrow [Ag^+] = 10^{-5} M$

No. of moles of Ag in 1 lit. solution = 10^{-5} mol

No. of moles of Ag in 250 ml solution $= \frac{10^{-5}}{4}$ mol

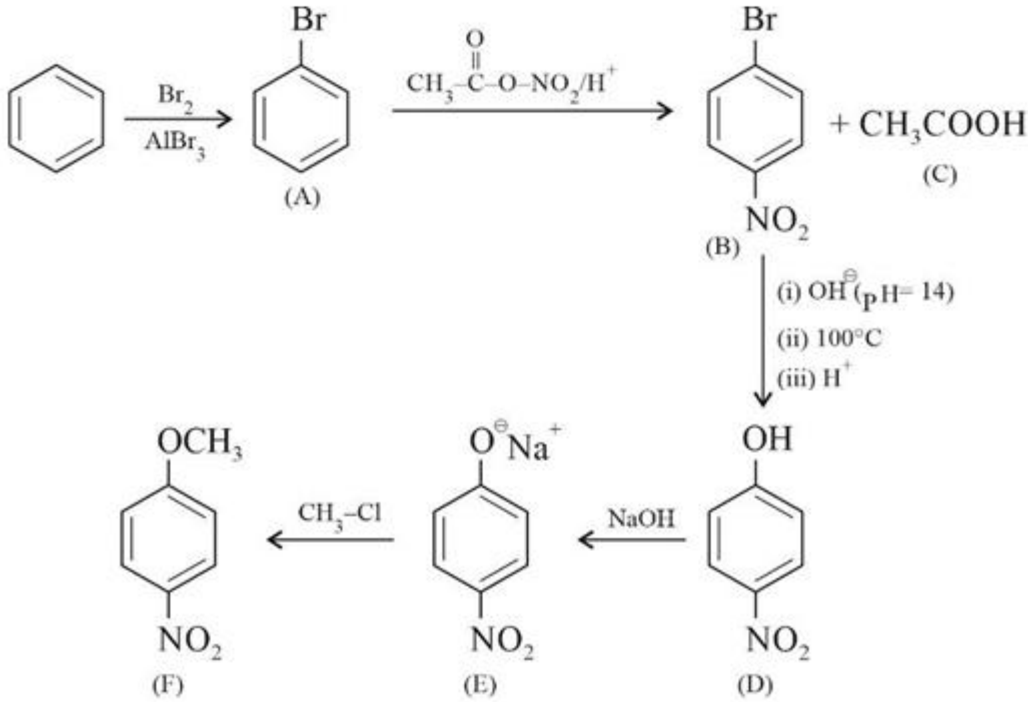
$w_{Ag} = \frac{10^{-5}}{4} \times 108 g = 0.27 mg$ % Pb in alloy $= \left(\frac{2.7 - 0.27}{2.7}\right) \times 100$ = 90%

3.(40) Distance between nearest neighbours $= 2r = \frac{a}{\sqrt{2}}$ $= \frac{4\sqrt{2}}{\sqrt{2}} = 4nm = 40A^\circ$

4.(75) % of S_N2 mechanism $= \frac{(6 \times 10^{-5})[R-X] \times 0.01}{\left[\left(6 \times 10^{-5}\right)[R-X] \times 0.01\right] + \left(2 \times 10^{-7}\right)[R-X]} \times 100 = \frac{6}{8} \times 100 = 75\%$

5.(5) Refer S_NAr reaction.

6.(153)


Br
Br$_2$
AlBr$_3$
(A)
CH$_3$–C(=O)–O–NO$_2$/H$^+$
Br
+ CH$_3$COOH
(C)
NO$_2$
(B)
(i) OH$^\ominus$ (pH= 14)
(ii) 100°C
(iii) H$^+$
OH
NO$_2$
(D)
NaOH
O$^\ominus$Na$^+$
NO$_2$
(E)
CH$_3$–Cl
OCH$_3$
NO$_2$
(F)

MATHEMATICS

SECTION – 1

1.(ABC) Equation $r(t) = R(u)$ we obtain $(1+t-2u)\hat{i} + (-10+2t-u)\hat{j} + (1+t-2u)\hat{k} = 0$

Thus $1+t-2u = 0; -10+2t-u = 0$

Solving, we obtain $t = 7, u = 4,$ *so* $\vec{r}(7) = 8\hat{i} + 8\hat{j} + 9\hat{k} = \vec{R}(4)$

The two lines intersect at the tip of this vector

2.(AC) $AB = \dfrac{1}{\left|(AB)^{-1}\right|} adj((AB)^{-1})$

$= |AB|\, adj(B^{-1}A^{-1})$

$= |A||B|\, adj(A^{-1})\, adj(B^{-1})$

3.(AC) $f(\theta) = \sin^3\theta + \cos^3\theta - \cos\theta\sin\theta(\sin\theta + \cos\theta)$

$= (\sin\theta + \cos\theta)^3 - 4\sin\theta\cos\theta(\sin\theta + \cos\theta) = (\sin\theta + \cos\theta)[1 - \sin 2\theta]$

Now $f(\theta) = 0$

$\Rightarrow \quad \tan\theta = -1$ or $\sin 2\theta = 1 \quad \Rightarrow \quad f(\theta) = 0$ has 2 real solutions in $[0, \pi]$

Also $\dfrac{f(\theta)}{1 - \sin 2\theta} = \sin\theta + \cos\theta$ and $\sin 2\theta \neq 1$; $\sin\theta + \cos\theta \in \left(-\sqrt{2}, \sqrt{2}\right)$

SECTION-2

4.(B) Equation of circle will be $x^2 + (y-2)^2 + \lambda(y-2) = 0$

Differentiating, we get $2x + 2(y-2)\dfrac{dy}{dx} + \lambda\dfrac{dy}{dx} = 0$

$\therefore$ the equation is $x^2 + (y-2)^2 - (y-2)\left(2x\dfrac{dx}{dy} + 2y - 4\right) = 0$

5.(A) Slope $= \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{y-1}{x^2+x}$

$\Rightarrow \quad \dfrac{dy}{y-1} = \dfrac{dx}{x^2+x} \quad \Rightarrow \int \dfrac{1}{y-1} dy = \int \dfrac{1}{x} - \dfrac{1}{x+1} dx + C \quad \Rightarrow \quad \dfrac{(y-1)(x+1)}{x} = k$

Putting $x = 1$, $y = 0$, we get $k = -2$ The equation is $(y-1)(x+1) + 2x = 0$

6.(A) $3\vec{i} + 2\vec{j} - 5\vec{k} = \lambda(2\vec{i} - \vec{j} + \vec{k}) + \mu(i + 3 - 2k) + \nu(-2\vec{i} + \vec{j} - 3\vec{k})$

$3 = 2\lambda + \mu - 2\nu$; $2 = -\lambda + 3\mu + \nu$; $-5 = \lambda - 2\mu - 3\nu$

$\Rightarrow \mu = 1$; $\nu = 2, \lambda = 3$

$\mu, \dfrac{\lambda}{2}, \nu$ are in A.P.

7.(A) $(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) = \vec{a} \times (\vec{a} \times \vec{b}) + \vec{b} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2 b + |\vec{b}|^2 \vec{a} - (\vec{a} \cdot \vec{b})b$

$= \vec{a}\vec{b}(\vec{a} - \vec{b}) - \vec{b} + \vec{a} \qquad (|\vec{a}| = |\vec{b}| = 1)$

$= ((\vec{a} \cdot \vec{b}) - 1)(\vec{a} - \vec{b})$

Hence $(\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})$ is parallel to $\vec{a}-\vec{b}$

SECTION-3

8.(D) **Let** $h(x)=f(x)-3g(x)$

$h(0)=-3$

$h(1)=-3$

$h(0)=h(1)$

9.(A) $f(2014)=f(2016)=f(2018)=0$

Let $g(x)=e^{-2x}f(x) \Rightarrow g(2014)=g(2016)=g(2018)=0$

As $f(x)$ is twice differentiable

$g(x)$ is also twice differentiable

$g'(x)=e^{-2x}\left(f'(x)-2f(x)\right)$

$g''(x)=e^{-2x}\left(f''(x)-4f'(x)+4f(x)\right)$

As $e^{-2x}\neq 0$ for any real x

We can prove option A, C, D by Role's mean value theorem.

$g'(x)=0$ for some $x\in(2014,\ 2016)$

$g'(x)=0$ for some $x\in(2016,\ 2018)$

i.e. $g'(x)=0$ has atleast two real roots

$\Rightarrow$ i.e. $g''(x)=0$ has one real roots

Similarly, $f'(x)$ has two roots and $f''(x)$ has one root.

10.(D) **11.(D)**

$$\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$$

$$\therefore \quad \cos\theta=\frac{xy}{6}-\sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}$$

$$\sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}=\left(\frac{xy}{6}-\cos\theta\right)$$

$$\left(1-\frac{x^2}{4}\right)\left(1-\frac{y^2}{9}\right)=\left(\frac{xy}{6}-\cos\theta\right)^2$$

$$\Rightarrow \quad 1-\frac{x^2}{4}-\frac{y^2}{9}+\frac{x^2y^2}{36}=\frac{x^2y^2}{36}+\cos^2\theta-\frac{xy\cos\theta}{3}$$

$$\Rightarrow \quad 1-\cos^2\theta=\frac{x^2}{4}+\frac{y^2}{9}-\frac{xy\cos\theta}{3} \quad \Rightarrow \quad 9x^2+4y^2-12xy\cos\theta=36\sin^2\theta$$

$$\therefore \quad N=36 \quad \text{and} \quad (\cos^{-1}x)^2-(\sin^{-1}x)^2>0$$

$$\Rightarrow \quad \frac{\pi}{2}\left(\cos^1 x-\sin^{-1}x\right)>0 \quad \Rightarrow \quad x\in\left[-1,\frac{1}{\sqrt{2}}\right)\equiv[p,\ q)$$

$\therefore \quad p = -1,\ q = \frac{1}{\sqrt{2}}$

10. $\sqrt{N} - 6 = \sqrt{36} - 6 = 0 \in \left[-1, \frac{1}{\sqrt{2}}\right)$

11. $\sec^{-1} x$ is not defined at $x = 0$

SECTION-4

1.(9) The total number of ways line can proceed is 6!

The total number of favourable ways $= \frac{^6C_3}{4} \cdot 3! \cdot 3!$

$p = \frac{1}{4} \quad \Rightarrow \quad 36p = 9$

2.(4) $I = \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x}\, dx = \int_0^{2\pi} \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x}$

$\Rightarrow \quad I = \int_0^{2\pi} \frac{\pi \sin^8 x}{\sin^8 x + \cos^8 x}\, dx = 4\int_0^{\pi/2} \frac{\pi \sin^8 x}{\sin^8 x + \cos^8 x} dx = 4\int_0^{\pi/2} \frac{\pi \cos^8 x}{\cos^8 x + \sin^8 x} = dx$

$\therefore \quad 2I = 4\pi \int_0^{\pi/2} \frac{\sin^8 x + \cos^8 x}{\cos^8 x + \sin^8 x}\, dx = 2\pi^2$

$\therefore \quad I = \pi^2$. Hence $k = 4$

3.(3) Area of quadrilateral $OABC = \Delta OAC + \Delta ABC$

$= \frac{1}{2}\left|\overrightarrow{OA} + \overrightarrow{AC}\right| + \frac{1}{2}\left|\overrightarrow{AB} + \overrightarrow{BC}\right| = \frac{1}{2}\left|\vec{a} \times (\vec{b} - \vec{a})\right| + \frac{1}{2}\left|(2\vec{a} + 10\vec{b} - \vec{a}) \times (\vec{b} - 2\vec{a} - 10\vec{b})\right|$

$= \frac{1}{2}\left|\vec{a} \times \vec{b}\right| + \frac{1}{2}\left|\vec{a} + 10\vec{b}\right| = 6\left|\vec{a} \times \vec{b}\right|$

$\left|\vec{a} \times \vec{b}\right| = m\ ; \quad \lambda = 2\lambda m \quad \Rightarrow \quad 6\left|\vec{a} \times \vec{b}\right| = 2\lambda\left|\vec{a} \times \vec{b}\right| \quad \Rightarrow \quad \lambda = 3$

4.(1) $\hat{i} \times \left[(\vec{a} - \hat{j}) \times \hat{i}\right] = (\hat{i} \cdot \hat{i})(\vec{a} - \hat{j}) - (\hat{i} \cdot (\vec{a} - \hat{j}))\hat{i} = \vec{a} - \hat{j} - (\hat{i} \cdot \vec{a})\hat{i}$

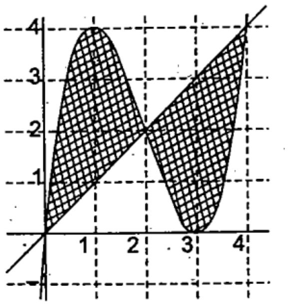
$\therefore \quad \vec{a} - \hat{j} - (\hat{i} \cdot \vec{a})\hat{i} + \vec{a} - \hat{k} + (\hat{j} \cdot \vec{a}) + \vec{a} - \hat{i} - (\hat{k} \cdot \vec{a})\hat{k} = 0$

$3\vec{a} - (\hat{i} + \hat{j} + \hat{k}) - \vec{a} = 0$

$\vec{a} = \frac{1}{2}(\hat{i} + \hat{j} + \hat{k}) = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \quad x = y = z = \frac{1}{2} \quad \Rightarrow \quad 8(x^3 - xy + zx) = 8(x^3 - x^2 + x^2) = 8 \times \frac{1}{8} = 1$

5.(8) Required area $= 2\int_0^2 \left(x(x-3)^2 - x\right) dx = 8$ sq. units



6.(10) $\vec{\alpha}\cdot\beta=\left(\frac{\hat{i}}{a}+\frac{4\hat{j}}{b}+b\hat{k}\right)\cdot\left(b\hat{i}+a\hat{j}+\frac{1}{b}\hat{k}\right)=\frac{b}{a}+\frac{4a}{b}+1$

$$\frac{1}{5+\alpha\cdot\beta}=\frac{1}{5+\frac{b}{a}+\frac{4a}{b}+1}=\frac{1}{\frac{b}{a}+\frac{4a}{b}+6}$$

Then the minimum value of $\frac{b}{a}+\frac{4a}{b}$ is 4 $\therefore$ Maximum value of $\frac{1}{5+\alpha\cdot\beta}$ is 0.10